fixed during the continuation procedure. For the application of the continuation method, it is necessary to know some starting point on the curve. In this study, a random search of such points is used.

3.3 Results for the Study of Tumbling

Figure 2 shows the trajectories of periodic rotations, describing stationary tumbling solutions in system (1) for different elevator deflections in the range $-25^\circ < \delta_e < 25^\circ$ with step 4° for the following parameters: flight altitude $H = 12\,000$ m, aircraft center of gravity displacement $\Delta x/\bar{c} = -0.05$ relative the basic value $x/\bar{c} = 0.275$, thrust value $T = 43.1$ kN. The thrust value corresponds to the level flight at this altitude with the total velocity $V = 100$ m/s. Figure 3 shows maximum and minimum values of these rotations and their period T depending on the elevator deflection. This figure shows also the similar results for center of gravity displacement $\Delta x/\bar{c} = -0.02$. The calculated rotations are stable only in the range of elevator deflections: 18°–25° at $\Delta x/\bar{c} = -0.05$ and 24°–25° at $\Delta x/\bar{c} = -0.02$ (they are shown by large markers).

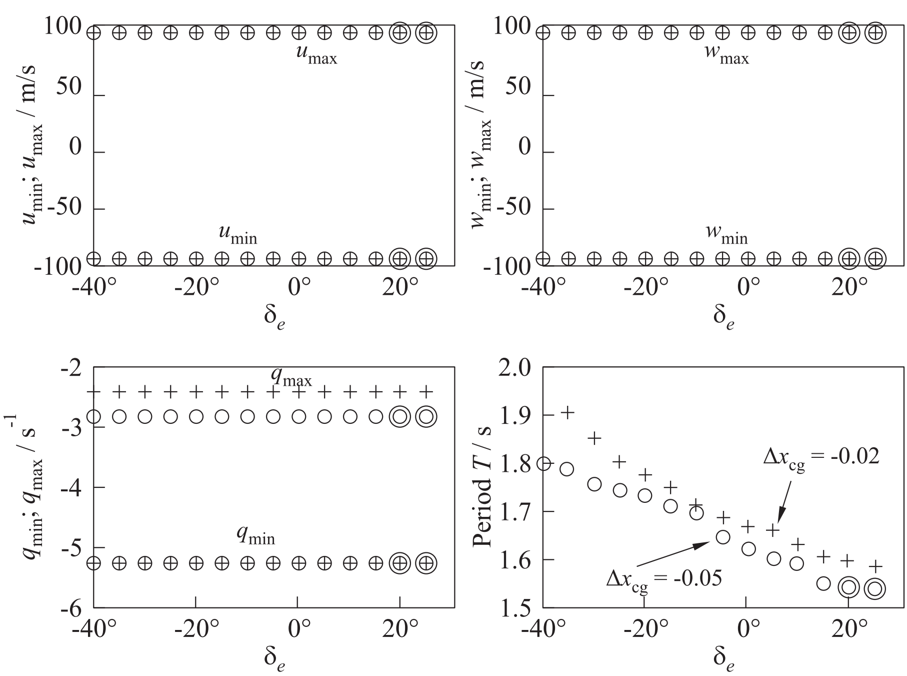

Figure 3 Maximum and minimum values of periodic rotation trajectories and their period depending on elevator deflection, $q < 0$

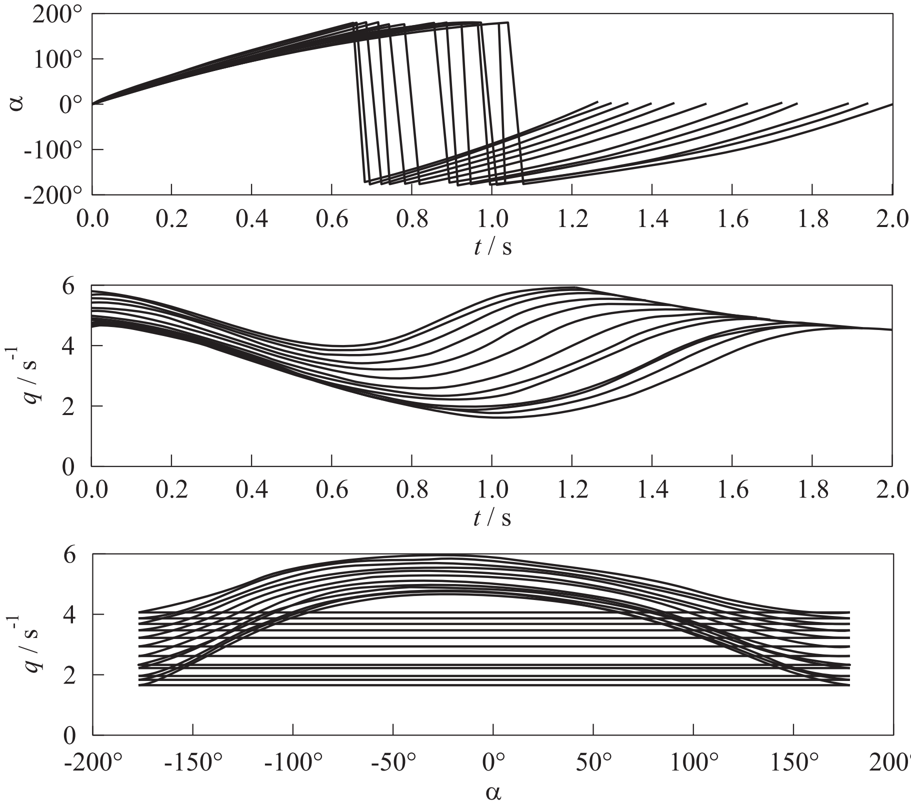

Figure 4 Periodic rotation trajectories depending on elevator deflection, $q > 0$

The parameter dependent boundary-value problem (4) has the similar solutions corresponding to rotations with pitch rate $q < 0$. Figure 4 shows α and q components of this solutions for the same grid in the parameter δ_e and the same flight parameters as in Fig. 2. All these periodic rotations are unstable.

The fact of instability of periodic solutions at most physically admissible parameters means that the considered aircraft has no tendency to tumbling. Nevertheless, large disturbances can lead to one or several aircraft turns. It is important to calculate the boundaries of such tumbling motion, i. e., to find the initial conditions for which at least one turn occurs. For estimation of the domains of tumbling, note that total velocity is practically constant during a turn. This allows considering this problem for a short-period approximation of the longitudinal motion.

To determine the parameters of tumbling onset, a grid in (α–q) plane with a sufficiently small step was used. Nodes of this grid were used as initial condi-

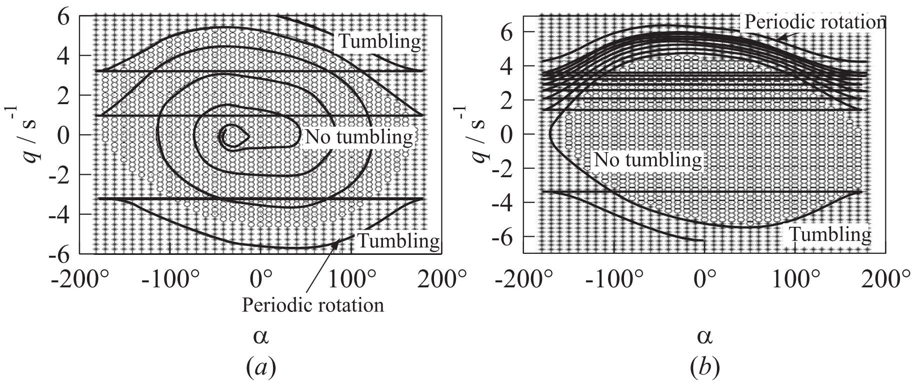

Figure 5 Domains of tumbling, stable periodic rotation, and example of trajectory originated in the region of tumbling at $H = 12$ km; $V = 100$ m/s; $\Delta x/\bar{c} = -0.05$; and $\delta = 0°$ (a) and $-30°$ (b)

tions for simulation of the short-period approximation of system (1). If the outgoing trajectory reaches angle of attack equal to $+180°$ or $-180°$ at least once, the point is marked as belonging to the tumbling domain. Otherwise, there is no tumbling. Figures 5 and 6 show the areas of tumbling for several values of the parameters: center of gravity position and elevator deflection. Several trajectories including trajectories of periodic rotations are also shown in these figures.

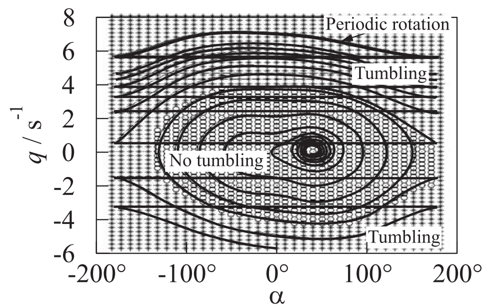

Figure 6 Domains of tumbling, stable periodic rotation, and example of trajectory originated in the region of tumbling at $H = 12$ km; $V = 100$ m/s; $\Delta x/\bar{c} = -0.1$; and $\delta = -30°$

Numerical simulation shows that regions tumbling absence in the (α–q) plane are convex. This allows to formulate the tumbling onset problem as follows: for each α value, find such a minimal (in absolute value) q that the boundary-value problem

$$\alpha(T) = \pi$$

has a solution and find such a minimal q that the boundary-value problem

$$\alpha(T) = -\pi$$

has a solution. The above problem formulation allows calculating the tumbling boundaries at a lower cost than the direct scanning. The results of application

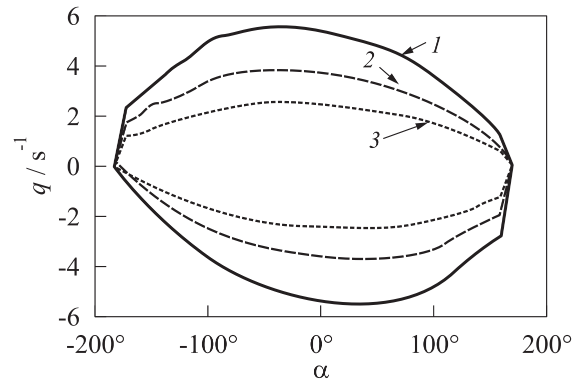

Figure 7 Tumbling boundaries depending on total velocity at H = 12 km; $\Delta x/\overline{c}$ = −0.05; and $\delta = 0°$: *1* — V = 110 m/s; *2* — 75; and *3* — V = 50 m/s

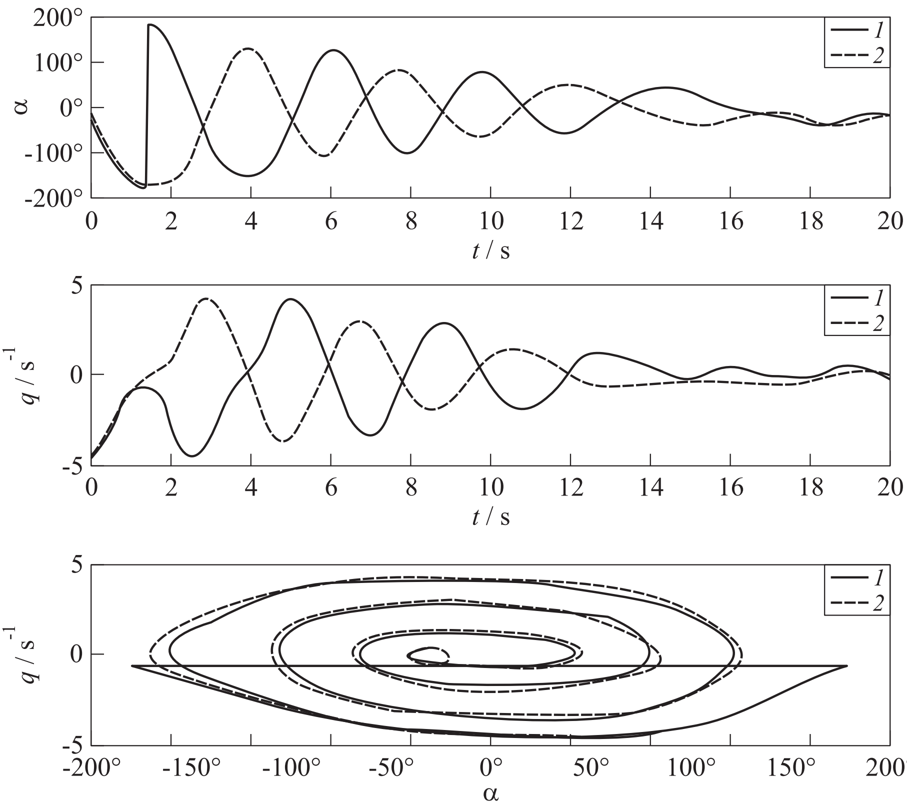

Figure 8 Trajectories near the tumbling boundary: *1* — $q(0) = 0.87q_{0\ \mathrm{circle}}$; and *2* — $q(0) = 0.86q_{0\ \mathrm{circle}}$

of this algorithm of computing the tumbling boundaries are shown in Fig. 7. Trajectories near the calculated tumbling boundary are illustrated in Fig. 8. Calculation of tumbling boundaries for the different aircraft and flight parameters has shown that the tumbling boundary moves to the less pitch rates with increasing the flight altitude, or decreasing the total velocity, or increasing forward the aircraft center of gravity position.

4 CONCLUDING REMARKS

The problem of the tumbling boundaries of a generic wing-only aircraft has been considered. With the use of continuation technique, periodic autorotation solutions have been calculated. The influence of flight altitude, total velocity, center of gravity position, elevator displacement, and initial conditions has been analyzed. The investigation has shown that the minimum initial pitch rate resulting in tumbling motion decreases with of flight altitude increasing, total velocity decreasing, center of gravity position forward displacement, and elevator positive deflection.

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