LOW ORDER $H_\infty$ OPTIMAL CONTROL FOR ACFA BLENDED WING BODY AIRCRAFT

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Advanced nonconvex nonsmooth optimization techniques for fixed-order $H_\infty$ robust control are proposed in this paper for design of flight control systems (FCS) with prescribed structure. Compared to classical techniques — tuning of and successive closures of particular single-input single-output (SISO) loops like dampers, attitude stabilizers, etc. — all loops are designed simultaneously by means of quite intuitive weighting filters selection. In contrast to standard optimization techniques, though ($H_2$, $H_\infty$ optimization), the resulting controller respects the prescribed structure in terms of engaged channels and orders (e.g., proportional (P), proportional-integral (PI), and proportional-integral-derivative (PID) controllers). In addition, robustness with regard to multimodel uncertainty is also addressed which is of most importance for aerospace applications as well. Such a way, robust controllers for various Mach numbers, altitudes, or mass cases can be obtained directly, based only on particular mathematical models for respective combinations of the flight parameters.

1 INTRODUCTION

The novel aircraft concepts and structures, like blended wing body (BWB) aircraft (Fig. 1, adopted from [1]), bring much more fuel efficiency and noise reductions but simultaneously several control design challenges. Rigid body motions as well as structure flexible modes appear in narrow frequency range, which requires more advance control techniques to avoid spillover of rigid-body motion into flexible modes (excitation of structural modes during aircraft manoeuvres) and vice versa. The flight dynamics, exhibiting several oscillatory or unstable modes for a typical aircraft, as well as the automatic or semiautomatic regimes of modern autopilots call for control synthesis methods that can effectively address these...
issues. Traditionally, classical tools for SISO loops tuning are used successively to deliver a complex FCS composed of a few smartly preselected channels, like pitch, roll or yaw dampers for suitable dynamics modifications (stability augmentation), subsequent attitude hold autopilots, automatic navigation loops, etc. [2,3]. Typically, a significant number of iterations and “back-stepping” is required as the higher-level loops interact partially with the lower-level pre-designed parts. Historically, frequency response methods were developed first in the 1930s and 1940s, and they remain the most commonly used methods till these days.

In this paper, a completely different approach towards this goal is suggested though. Thanks to practical availability of CACSD (Computer Aided Control Systems Design) tools based on most recent nonconvex nonsmooth optimization techniques, direct synthesis methods can be employed to deliver a complex FCS that is structured (features preselected channels only), of fixed low-order (consisting, for example, of P, PI, lead-lag controllers), optimal in the $H_\infty$ norm sense (for bandwidth setting, reference tracking, disturbance attenuation requirements), and robust with regard to multimodal uncertainty (covering a selected number of airspeed, mass, altitude, or other cases) [2,4,5].

2 CONTROL LAW DESIGN APPROACH — FIXED-ORDER OPTIMIZATION

First of all, it is necessary to point out that there are more tools for fixed-order optimization (like hinfsstruct Matlab toolbox, presented on IFAC World Congress 2011). The aim of this paper is not the comparison or evaluation of several approaches, as the authors are clearly not unbiased users of such tools.
In order to directly obtain a robust feedback controller of prespecified order, the $H_{\text{inf}}$ Fixed-Order Optimization (HIFOO) toolbox is used, outlined in detail in [6,7]. The HIFOO control design method searches for locally optimal solutions of a nonsmooth optimization problem that is built to incorporate minimization objectives and constraints for multiple plants (Fig. 2). The controller structure is fixed at the outset, allowing for structured controller design. The resulting optimization problem is formulated on the controller coefficients only, resulting in a typically small-dimensional nonsmooth nonconvex optimization problem that does not require the solution of large convex subproblems, relieving the computational burden typical for Lyapunov LMI (linear matrix inequality) techniques. Because finding the global minimum of this optimization problem may be hard, an algorithm that searches only for local minima is used. While no guarantee can be given on the result quality of this algorithm, in practice, it is often possible to determine a satisfying controller with a reasonable computational effort.
3 LONGITUDINAL CONTROL

3.1 Model Description

Longitudinal flight mechanics and aeroelastic effects of a large BWB aircraft design and their coupling were modeled in an integrated form for the purpose of control system design as well as validation.

In this section, the longitudinal dynamics is considered to design control law for the longitudinal motion. A set of linearized state space systems for various parameter values of fuel and payload mass (at fixed cruise altitude and airspeed) are available:

\[ \dot{x} = Ax + Bu; \quad y = Cx + Du \]

where the state vector \( x \) is composed of 6 flight-mechanic states (\( x \)-position \( X \), body forward speed \( u \), altitude \( Z \), body down speed \( w \) (it is proportional to the angle of attack \( \alpha \)), pitch angle \( \theta \), and pitch rate \( q \)), 12 elastic states (6 symmetrical structural modes), as well as 7 aerodynamic lag states. The states \( X \) (\( x \)-position) and \( Z \) (altitude) are neglected in this study and are removed from the system description for control law design and validation.

Utilized inputs \( u \) for control design are:

- Symmetric elevator (El) and beaver tail (BT) deflection and rate (those control surfaces are actuated simultaneously).

The two different actuator dynamics models are involved. The first one is the complex actuator model used for control law validation. The complex model consists of the third-order linear dynamic description (electrical driver, mechanical part, and rate to deflection integrator), first-order LQI (linear quadratic integral) control law, nonlinear constrains of actuator dynamic (saturation and rate limit), and, finally, the description of atmospheric influence. Next, the complex “validation” model was linearized to receive suitable mathematical model for control law design, modeled via second-order low-pass filters.

Utilized outputs \( y \) for control design are:

- pitch rate \( q \); and
- normal acceleration \( N_z \)

where in both sensor signals, 160-millisecond time delay (due to signal processing latency, modeled via a second-order Padé approximation) and low-pass Butterworth filters of the second order were considered. The overall design model is of order 35 and the validation model order is even higher (minimum order needed for validation is 39).
3.2 Control Law Design

The lateral Control Augmentation System (CAS) of extremely low order (first-order control law) with imprint structure was design by HIFOO toolbox (Fig. 3). The structure of the control law is shown in Fig. 4. It is a commonly used hierarchical control law used for an asymptotic tracking of the aircraft normal acceleration reference signal.

The hierarchical control law design was usually done in the iterative manner, using background knowledge of the physical meaning of the single loop to

![Diagram of control law design](image)

**Figure 3** Longitudinal CAS (a) and longitudinal CAS with flexible modes damper (b)

![Diagram of control law with structure](image)

**Figure 4** Longitudinal control law with structure
reach required performance. The optimization technique is addressed now to design the overall control law in one shot. An $H_{\infty}$ performance criteria can be introduced to design robust control law with predefined structure and order. The extremely low order and structural complexity of overall control law (with preserved robust behavior and control performance of full multiple-input multiple-output (MIMO) high-order control laws) is very important for final onboard implementation. It reduces necessary computational effort and, therefore, hardware demands for onboard equipment, which is closely connected with reliability and price of implementation. For other possibilities of CAS designs, see [8–11].

The generalized plant used for longitudinal control law design is plotted in Fig. 5 where the aircraft dynamic is described by plant $G$ with state vector, input, and output according to subsection 3.1. The actuators models are already included in aircraft model, but the sensors models are presented separately by Sensors block.

The criterion represented by weighting filters is presented in Fig. 5 by the $W$ block. The used weighting filters shapes are presented in Fig. 6. The low pass filter (Fig. 6a) is usually used to define performance of control law. The band pass filter (Fig. 6b) is usually used for oscillatory modes damping and eventually, the high pass filter (Fig. 6c) is usually used as an actuator constrains in a frequency domain.

The weightings filters are usually of first order (second order in the case of band pass filter) to keep design plant order low. The overall order of augmented plant is 45.
3.3 Results and Simulations

In this subsection, the resulting longitudinal control law performance is presented. The control goals are defined as follows:

- the required closed loop poles relative damping need to be less or equal to 0.5 for all rigid body poles;
- the phugoid mode can be even unstable, but real and with time constant less than 0.1; and
- the Nz step response rise time needs to be less than 7 s.

The closed loop poles locations can be seen in Fig. 7.

The aircraft normal acceleration step response can be seen in Fig. 8 where the design plant (without phugoid mode) response as well as the validation plant (with phugoid mode) responses are plotted for all fuel cases (which is one of the robust behavior requirements).
Figure 7  Poles and zeros location (design (a) and validation (b)) of BT + El to Nz transfer function. All fuel cases are plotted for open loop (1) and closed loop (2).

Figure 8  Nz reference signal tracking for all fuel cases. Design plant (without phugoid mode) and validation plant (with phugoid) are potted. Axes descriptions are hidden because of confidential reasons.
Eventually, the robustness of control law with respect to unmodeled input and output uncertainty is presented. The uncertainty is here illustrated by diamonds in a Nichols charts representing phase lag and change of open loop magnitude on actuators or sensors side (here considered to occur simultaneously). One Nichols chart is used for each loop of multiple inputs and single output control law to validate controller robustness. There are different robustness requirements for predefined frequency regions of control law, bounded by phugoid mode frequency, short period mode frequency, and the first wing bending mode frequency.

Each robustness requirement is defined by different size of diamonds in Nichols chart (solid, dash-dotted, and dotted lines). The Nichols diamonds violation presented in Figs. 9 and 10 is not violation of robustness criteria as it occurs at frequency range lower than is the frequency of phugoid mode, short period mode, or first wing bending mode, respectively.

4 FLEXIBLE MODES CONTROL

4.1 Model Description

Analogous to the longitudinal case, the lateral flight mechanics and aeroelastic effects of a large BWB aircraft design and their coupling were modeled in an integrated fashion. In this study, the lateral dynamics is considered by the authors to design control laws for the lateral motion. A set of linearized state space systems for various parameter values of fuel and payload mass (at fixed cruise altitude and airspeed) are available:

\[ \dot{x} = Ax + Bu; \quad y = Cx + Du \]
where the state vector $\mathbf{x}$ is composed of 6 flight-mechanic states ($y$-position $Y$, body side velocity $v$ (proportional to side slip angle $\beta$), roll rate $p$, yaw rate $r$, roll angle $\Phi$, and yaw angle $\Psi$), 12 elastic states (6 antisymmetrical structural modes), as well as 7 aerodynamic lag states. The states $\Psi$ (yaw angle) and $y$ (horizontal displacement) are neglected and removed in this study.
Utilized inputs $u$ for control design are:
- combined rudder deflection and rate; and
- two antisymmetric flaps deflections and deflection rates: outer and inner flaps.

The two different actuator dynamics models are involved. The first one is the complex actuator model used for control law validation. The complex model consists of the third-order linear dynamic description (electrical driver, mechanical part, and rate to deflection integrator), first-order LQI control law, nonlinear constraints of actuator dynamic (saturation and rate limit), and, finally, the description of atmospheric influence. Next, the complex "validation" model was linearized to receive suitable mathematical model for control law design, modeled via second-order low-pass filters.

Utilized outputs $y$ for control design are:
- side slip angle $\beta$;
- roll angle $\Phi$;
- yaw rate $r$; and
- roll rate $p$.

where in all sensor signals 160-millisecond time delay (due to signal processing latency, modeled via a second-order Padé approximation) and low-pass Butterworth filters of the second-order were considered. The overall design model is of order 41 and the validation model order is even higher (minimum order needed for validation is 47).

4.2 Control Law Design

The lateral integrated CAS was designed as a two-degree-of-freedom (2DoF) architecture using fixed-order optimization approach to keep control law order low, but any structure of the control law is not required in this case. The resulting extremely low-order (in this case, the third-order control law was designed) controller was built using HIFOO toolbox. Overall lateral CAS consist of RB autopilot (roll and beta tracker with Dutch roll damper). The lateral CAS setup can be seen in Fig. 11. Two reference signals are used as inputs into feedforward part of controller (roll and beta setpoints). The beta reference signal is usually set to zero and then CAS provides coordinated turn functionality. Control surfaces used by CAS are two flaps (antisymmetrically actuated) and rudders (symmetrically actuated). Measured signals are lateral RB variables at CG ($\beta$, $\Phi$, $p$, and $r$). Lateral control law was designed as a fully integrated lateral MIMO control law without imprint structure. The actuators limits like saturations and rate limits were considered during the design process.
The generalized plant used for lateral control law design is plotted in Fig. 12 where the aircraft dynamic is described by plant $G$ with state vector, input, and output according to subsection 4.1. The actuators models are already included in aircraft model, but the sensors models are presented separately by Sensors block.

The criterion represented by weighting filters is in Fig. 5 are presented by the $W$ block. The overall order of augmented plant is 57.

### 4.3 Results and Simulations

First, roll maneuver was investigated in Fig. 13a for all fuel case at cruise case. The side slip angle reference signal tracking is shown in Fig. 13b. The Dutch roll mode damping in time as well as frequency domain are shown in Fig. 14.
Figure 13 Integrated lateral $H_{\infty}$ optimal CAS: (a) roll angle and rate response for all fuel cases; and (b) side slip angle response

Figure 14 Dutch roll damper demonstration. Rudder to yaw rate step response (a) and Bode plot (b): 1 — open loop and 2 — closed loop. Axes descriptions are hidden because of confidential reasons
5 CONCLUDING REMARKS

The novel approach for lateral and longitudinal control systems were presented, both designed with respect to simplicity of the controller structure and low-order requirements. Imprint structure of control law was presented for longitudinal control law, still designed by optimization techniques in one shot. Full structure, but of low order control law was presented for lateral control. Requirements for performance as well as robust behavior were fulfilled for both presented control laws.

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